MATHEMATICS STANDARDS

1. Uses a variety of strategies in the problem-solving process

2. Understands and applies basic and advanced properties of the concepts of numbers

3. Uses basic and advanced procedures while performing the processes of computation

4. Understands and applies basic and advanced properties of the concepts of measurement

5. Understands and applies basic and advanced properties of the concepts of geometry

6. Understands and applies basic and advanced concepts of statistics and data analysis

7. Understands and applies basic and advanced concepts of probability

8. Understands and applies basic and advanced properties of functions and algebra

9. Understands the general nature and uses of mathematics

Fig. 1

MATHEMATICS STANDARD AND BENCHMARKS

Standard 8: Understands and applies basic and advanced properties of functions and algebra

Level I (Grade K-2)

1. Recognizes regularities in a variety of contexts (e.g., events, designs, shapes, sets of numbers)

2. Extends simple patterns (e.g., of numbers, physical objects, geometric shapes)

Level II (Grade 3-5)

1. Recognizes a wide variety of patterns (e.g., basic linear patterns such as [2,4,6,8.. simple repeating and growing patterns) and the rules that explain them

2. Understands that the same pattern can be represented in different ways (e.g., geometrically or numerically; the pattern of numbers [7,14,21,28..] is equivalent to the mathematical relationship $7 \times n$)

3. Knows that a variable is a letter or symbol that stands for one or more numbers

4. Understands the basic concept of an equality relationship (i.e., an equation is a number sentence that shows two quantities that are equal)

5. Solves simple open sentences involving operations on whole numbers (e.g., ? + 17= 25)

6. Knows basic characteristics and features of the rectangular coordinate system (e.g., the horizontal axis is the X axis and the vertical axis is the Y axis)

Level III (Grade 6-8)

1. Knows that an expression is a mathematical statement using numbers and symbols to represent relationships and real-world situations (e.g., equations and inequalities with or without variables)

2. Understands that a variable can be used in many ways (e.g., as a placeholder for a specific unknown, such as $x + 8 = 13$; as a representative of a range of values, such as $4t + 7$)

Fig. 2

3. Understands various representations (e.g., tables, graphs, verbal descriptions, algebra expressions, Venn diagram) of patterns and functions and the relationship among them

4. Understands the basic concepts of a function (i.e., functions describe how changes in a quantity or variable result in changes in another)

5. Solves linear equations using concrete, informal, and formal methods (e.g., using properties, graphing ordered pairs, using slope-intercept form)

6. Solves simple inequalities and non-linear equations with rational number solutions, uses concrete and informal methods

7. Understands special values (e.g., minimum and maximum values, x-and y-intercepts constant ratio or difference) of patterns, relationships and functions

Fig. 2 (cont'd)

Critical Questions Used for Determining
Knowledge Domain Classification of Standards

1. Does the standard contain generalizations, principles, or overarching ideas and/or what examples will be used to support these?

If yes, then Declarative   If no, then Procedural

2. Does the standard contain essential time sequences, cause/effect sequences, or episodes that students will need to remember and use at a later date?

If yes, then Declarative   If no, then Procedural

3. Does the standard contain essential vocabulary terms or phrases that would be important for students to learn?

If yes, then Declarative   If no, then Procedural

4. Does the standard contain any processes that students will need to practice, and what are the subcomponents of these processes?

If yes, then Procedural   If no, then Declarative

5. Does the standard contain any skills (tactics or algorithms) that students will need to practice in order to gain proficiency and what are the steps or rules that students will need for these skills?

If yes, then Procedural   If no, then Declarative

6. Does measurement of the standard require the use of explicit examples, relationships, and the absence of misconceptions (in those examples)?

If yes, then Declarative   If no, then Procedural

7. Does measurement of the standard require a performance that is carried out with ease, a level of automaticity, and without error?

If yes, then Procedural   If no, then Declarative

If critical attributes of BOTH Declarative and Procedural Knowledge are contained in the Standard, categorize it as "Both."

Fig. 3

Examples of benchmarks that represent Declarative Knowledge

Understands the properties and theorems of roots, exponents, and logarithms.

Understands that words and pictures convey ideas or meaning in a text.

Understands that animals have characteristics that help them adapt to their environment.

Knows the causes and effects of the American Revolution.

Know the rules that govern various sports.

Understands the concept of mutation.

Knows that when oppression meets resistance, the result is often conflict.

Fig. 4

Examples of benchmarks that represent Procedural Knowledge

Solves multi-step problems involving fractions, decimals, and basic percents.

Uses prewriting strategies to plan written work.

Predicts possible results of scientific investigations.

Solves simple inequalities and non-linear equations with rational number solutions, using concrete and informal methods.

Summarizes information found in texts.
Uses locomotor skills in rhythmical patterns (e.g., even, uneven, fast, and slow).

Uses conjunctions in written compositions.

Fig. 5

Critical Questions Used for Determining
Brain Processing Functions in the System of Thinking
for Knowledge Acquisition

NOTE: ALL 3 SYSTEMS OF THINKING (Self, Metacognitive, Cognitive) MUST BE ENGAGED FOR LEARNING TO OCCUR. CHOOSE THE PRIMARY SYSTEMS AND FUNCTIONS ENGAGED FOR STUDENTS TO LEARN THE SPECIFIC KNOWLEDGE OF THE BENCHMARK. MORE THAN ONE SYSTEM AND FUNCTION MAY BE REQUIRED.

If YES, Self System if Required

1. Does the content knowledge require students to examine personal beliefs, self-attributes, purpose, and efficacy to learn the presenting task?

2. Does the content knowledge require students to examine whether or not they have the prerequisite skills to learn the presenting task?

3. Does the content knowledge require students to examine whether they are motivated to learn the presenting task?

If YES, Metacognitive System is Required

1. Does the content knowledge require students to set goals to learn the presenting task? (Goal Specification Function)

2. Does the content knowledge require students to retrieve and activate skills, tactics, and processes from procedural memory, and to sequence the order in which the skills, tactics, and processes will be used to learn the presenting task? (Process Specification Function)

3. Does the content knowledge require students to monitor or evaluate the effectiveness of their choices in learning strategies in terms of time needed and resources required to learn the presenting task? (Process Monitoring Function)

4. Does the content knowledge require students to monitor or evaluate the accuracy, clarity, restraint, intensity of task agreement, or task focus with which they approached the learning task? (Disposition Monitoring)

If YES, Cognitive System – Storage/Retrieval Function is Required

Fig. 6

1. Does the content knowledge require students to <u>embed or store knowledge</u> into long-term memory for later retrieval to address the presenting task?

2. Does the content knowledge require students to <u>retrieve or recall or activate knowledge</u> stored in long-term memory to address the presenting task?

If YES, Cognitive System – Information Processing Function is Required

1. Does the content knowledge require students to compare and contrast data in working memory to learn the presenting task? (Matching Processing)

2. Does the content knowledge require students to <u>represent data</u> in working memory (from the senses) in another format so that it can be embedded into long-term memory to learn the presenting task? (Idea Representation Processing)

3. Does the content knowledge require students to determine or <u>evaluate the reasonableness of data or the logical presentation of data</u> in order to compare the new data with known data to learn the presenting task? (Information Screening Processing)

4. Does the content knowledge require students to <u>generate inferences</u> (using inducting thinking) about specific information to learn the presenting task? (Information Generalization Processing)

5. Does the content knowledge require students to <u>categorize, associate, or examine cause/effect and temporal order relationships</u> (using deductive thinking) to learn the presenting task? (Information Specification Processing)

6. Does the content knowledge require students to <u>construct new knowledge by generating new propositions</u> using information from long-term memory in order to learn the presenting task? (Idea Production Processing)

If YES, Cognitive System – Input/Output Function is Required

1. Does the content knowledge require students to <u>decode information</u> presented orally (listening) or in visual form (reading) using phonemic, syntactic, and semantic features to make meaning in order to learn the presenting task?

Fig. 6 (cont'd)

2. Does the content knowledge require students to encode information retrieved from long term memory to create meaning in a visual form (writing) or an oral format (speaking) in order to learn the presenting task?

3. Does the content knowledge require students to engage in the <u>heuristics for listening</u>: a) decoding of oral language, b) analyzing the data in working memory (via the information processing functions), c) activating prior knowledge relative to the topic (via the retrieval function), and d) activating of knowledge (via the retrieval function) about the overall process of listening in order to learn the presenting task?

4. Does the content knowledge require students to engage in the <u>heuristics for reading</u>: a) decoding of written language (words, phrases, sentences), b) analyzing data in working memory (via the information processing function), c) activating (via retrieval function) prior knowledge about the topic, d) activating (via retrieval function) of prior knowledge about the type of discourse, and d) activating (via retrieval function) knowledge about the overall process of reading in order to learn the presenting task?

5. Does the content knowledge require students to engage in the <u>heuristics for speaking</u>: a) activating (via retrieval function) prior knowledge about the topic, b) activating (via retrieval function) knowledge about the overall process of speaking, c) analyzing data in working memory (via information processing function), and d) encoding the ideas generated in working memory into surface-level language in order to learn the presenting task?

6. Does the content knowledge require students to engage in the <u>heuristics for writing</u>: a) activating (via the retrieval function) prior knowledge about the topic, b) activating (via the retrieval function) knowledge about the type of discourse to be used, c) activating (via the retrieval function) knowledge about the overall writing processing function), and e) encoding of idea generation in working memory into surface-level language in order to learn the presenting task?

If YES, Cognitive System – Knowledge Utilization Function is Required

1. Does the content knowledge require students to make a <u>decision between two or more alternatives</u> in order to learn the presenting task? (Decision-making)

Fig. 6 (cont'd)

2. Does the content knowledge require students to <u>solve a problem</u> encountered when attempting to accomplish a goal in order to learn the presenting task? (Problem solving)

3. Does the content knowledge require students to <u>generate and test hypotheses</u> to understand some physical or psychological phenomenon with rules of evidence that require <u>statistical hypotheses testing</u> in order to learn the presenting task? (Experimental inquiry)

4. Does the content knowledge require students to <u>generate and test hypotheses</u> about past, present, or future events with rules of evidence that require sound <u>argumentation</u> in order to learn the presenting task? (Investigation)

Fig. 6 (cont'd)

Examples of Benchmark Knowledge Addressed by the <u>Self Sytem</u>

Selects reading material based on personal criteria

Makes connections between characters or simple events in a literary work and people or events in his or her own life

Relates new information to prior knowledge and experience

Understands that science and mathematics operate under common principles: belief in order, ideals of honesty and openness, the importance of review by colleagues, and the importance of imagination

Uses discussions with teachers and other students to understand problems

Fig. 7

Examples of Benchmark Knowledge Addressed by the <u>Metacognitive System</u>

Understands that some ways of representing a problem are more helpful than others

Selects and uses appropriate computational method

Adds, subtracts, multiplies, and divides whole numbers and decimals

Establishes a purpose for reading

Monitors own reading strategies and makes modifications as needed

Fig. 8

Examples of Benchmark Knowledge Addressed by the
Cognitive System – Storage and Retrieval Function

Counts whole numbers

Draws pictures to represent problems

Uses graphic organizers to gather and record information for research topics (e.g., notes, charts, graphs)

Represents problem situations in and translates among oral, written, concrete, pictorial, and graphical forms

Identifies and uses the various parts of a book (index, table of contents, glossary, appendix) to locate information

Understands common terms used with estimation (e.g., "about," "near," "closer to," "between," "a little less than")

Fig. 9

Examples of Benchmark Knowledge Addressed by the
Cognitive System – Information Processing Function

Understands the characteristics and properties (e.g., order relations, relative magnitude, base-ten place values) of the set of rational numbers and its subsets (e.g., whole numbers, fractions, decimals, integers)

Summarizes and paraphrases complex, implicit hierarchic structures in information texts, including the relationships among the concepts and details in those structures

Generalizes from a pattern of observations made in particular cases, makes conjectures, and provides supporting arguments for these conjectures (i.e., uses inductive reasoning)

Constructs informal logical arguments to justify reasoning process and methods of solutions to problems (i.e., uses informal deductive methods)

Analyzes the effectiveness of complex elements of plot

Makes connections among literary works based on theme

Fig. 10

**Examples of Benchmark Knowledge Addressed by the
Cognitive System – Input/Output Function**

Writes in response to literature

Decodes unknown words using basic elements of phonetic analysis and structural analysis

Applies reading skills and strategies to a variety of literary passages and texts

Responds to questions and comments (e.g., gives reasons in support of opinions)

Uses explicit techniques for oral presentation (e.g., modulation of voice, inflections, tempo, enunciation, physical gestures, eye contact, posture)

Listens to and understands the impact of nonprint media on media consumers (e.g., persuasive messages and advertising in media, the presence of media in people's daily lives, the role of media in forming opinions, media as a source of entertainment and information)

Fig. 11

**Examples of Benchmark Knowledge Addressed by the
Cognitive System – Knowledge Utilization Function**

Determines the effects of addition, subtraction, multiplication, and division on size and order of numbers

Formulates a problem, determines information required to solve the problem, chooses methods for obtaining this information, and sets limits for acceptable solutions

Synthesizes information from multiple research studies to draw conclusions that go beyond those found in any of the individual studies

Identifies and analyzes the philosophical assumptions and basic beliefs underlying an author's work

Uses a variety of reasoning process (e.g., reasoning from a counter example, underestimate, range of estimates) to solve real-world problems

Fig. 12

EXAMPLES OF REJECTED INSTRUCTIONAL STRATEGIES

<u>Standard</u>: Draws conclusions and makes inferences based on explicit and implicit information in texts.

	Reason for Rejecting	
<u>Strategy</u>	<u>Wrong Domain</u>	<u>Wrong Brain Process</u>
Metacognitive Problem Solving		X
Concept Attainment	X	X
Task Analysis – Problem Solving		X
Contextual Engaged Learning – Relating		X
Evaluative Critics	X	X
Model Making		X
Anticipation Guide	X	X
Explicit Instruction	X	X
Peer Response Groups		X
Manipulatives		X
Patterning	X	X
Synectics	X	X
Mneumonics	X	X
Peer Teaching		X
Computer Tutorials	X	X
Phonological Awareness	X	X
Semantic Mapping	X	X
Simulations	X	X
Pre and Post Writing Samples	X	
Authentic Assessment		X
Vocabulary Through Context	X	X
Writing Quality Feedback		X
Concept Formation – Inductive Thinking	X	X
Audience Oriented Communication		X
Designing Games		X
Writing for an Audience		X
Oral Reading for a Purpose		X
Phonics Plus Context	X	X
Constructivist Teaching		X
Authentic Problem Based Learning		X
Question Based Decision-Making – PLAN		X
Character Quotations		X
Brainstorming	X	X
Promoting Classroom Discourse		X
Writing Process – Primary Traits		X
Collaborative Strategy Instruction		X

Figure 13

IMPROVE Model for comprehension		X
Service Learning		X
Meaning Schemes	X	X
Phonemic and Syllable Instruction	X	X
P.L.A.N.		X
Guided Imagery	X	X
Think Time/Wait Time		X
ARCS for Motivation		X
Fostering Parallel Thinking		X
Case Studies		X
Monolithic Model of instruction		X
Mastery Learning	X	X
Concept Focused Instruction	X	X
Memorization of the Facts	X	X
Response Cards	X	X
Polling	X	X
Predictions		X
Music and Mathematics		X
Introspection Journals		X
Chunking	X	X

Figure 13   EV 069689854 US

EXAMPLES OF ALIGNED INSTRUCTIONAL STRATEGIES

<u>Standard</u>: Draws conclusions and makes inferences based on explicit and implicit information in texts:

Cues and Feedback
Verbalization
Linguistic Representations
Teaching Writing
Reviewing Knowledge, Interpretation and Judgment Questions
Vocabulary Strategies to Improve Comprehension
Responsivity to Student Contributions
Activating of Prior Knowledge
Content Comparisons
Goal Specification
Process Monitoring
Disposition Monitoring
Summarizing
Discourse Characteristics
Graphic Outlining
Learning Logs
Writing as a Response to Reading
Differentiated Instruction
Extension of Background Knowledge
Self-Directed Learning
Self-Evaluation of Learning
Concept Mapping
Academic Controversy
Creative Learning
Metacognitive Strategies for Writing/Reading
Teaching for Relevancy
Teaching for Transfer
Critical Reading Techniques
Task-Completion Expectations
Natural Acquisition of Language
Information Literacy
Differentiated Curriculum
Providing Examples and Non-Examples
Performance-Based Instruction
Steps of Processing Literature
Utilizing Reference Materials
Improving Content Reading and Writing Skills
Reflective Questioning
Effective Questioning Techniques

Figure 14

Open Discussion
Critical Thinking Skills – Authentic Learning
Elicitation of Bases for Statements of Positions
Reading and Writing for Meaning
Reflective Learning
Cooperative Learning (General)
Role Playing
Discussion and Analysis
Concrete to Abstract Learning
Thematic Reading/Writing/Speaking
Interactive Learning
Self-Directed Learning – Inner Speech
Self Directed Inquiry
Self-Directed Learning – Functional Failure
Critical Thinking Competencies
Meta-Comprehension
Primary Sources
Developing Oral Language/Student Centered
Pattern and Organization
Metacognitive Reading Perspectives
Oral Pre-reading Strategies
Building Categories for Critical Thinking
Cognitive-Metacognitive Combinations
Problem Solving
Mind Mapping for Skill Review
Heuristics
Multidisciplinary Integration
Engaged Learning

Figure 14